Half-life

In practical use, the duration of radioactive decay is defined with half-life ($t_{1/2}$) instead of the decay constant (λ). Half-life is the time taken for the activity of a given amount of a radioactive substance to decay to half of its initial value (in t_0 time). The connection between half-life and the decay constant is:

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0{,}693}{\lambda}$$

The measurement of isotopes with very long and very short half-life is difficult. By long half-life isotopes, the amount and activity of the active sample should be measured, and from these the decay constant and the half-life can be determined:

$$A = \frac{-dN}{dt} = \lambda N$$

where: A is the activity of the sample

$\frac{-dN}{dt}$ = the number of active particles differenced by time.

Measurements in the medium half-life range can be described with the following groups (t_m = is the length of measurements):

1. $t_{1/2} \langle\langle t_m$
2. $t_{1/2} \approx t_m$
3. $t_{1/2} \rangle\rangle t_m$

In the first case the half-life is short, therefore the activity of the radioactive material changes significantly during the measurement time. The statistical deviation cannot be reduced by parallel measurements.

If the half-life is much longer than the measurement time, the mathematical averaging of the results of parallel measurements can reduce the error. When the half-life is extremely long we face another problem because of the long measurement times, the errors originating from the instability of the measuring devices.

If the half-lives of the samples are from a few minutes to a few months, the half-life can be determined by measuring the activity (counts) of the sample at different times. The results can be used in the negative exponential equation of the radioactive decay, which can be described

with using the number of active nuclides, activity or – under proper conditions – the count rate.

$$N_t = N_0 e^{-\lambda t}$$

Where: N_0 = the number of active nuclides at t_0

N_t = the number of active nuclides after t time

t = time elapsed between N_t and N_0 measurements

Since the determination of the active nuclide's number and the activity is in most cases hard, usually the counts are used in calculations. This is only applicable when the position of the detector and the sample is fixed to each other (the geometry of the measurement does not change during experiments) and constant detector efficiency is ensured (fixing the detector voltage).

Then, the following equation can be used:

$$I_t = I_0 e^{-\lambda t}$$

where: I_0 = counts at t_0

I_t = counts after t time

Therefore if we measure the counts of the radioactive sample at two different times and the elapsed time, the decay constant and the half-life can be determined.

In many cases we have to count with the influence of background-radiation, primarily caused by particles emitted by the decay of cosmic and ground originated radionuclides. The effects can be reduced by proper shielding (lead tower), however it cannot be fully eliminated. Correction with background is required during calculations when the signal-noise ratio is worse than 100:1.

For laboratory experiments a Pu-Be laboratory neutron source is used for activating the inactive samples.

The operation of the Pu-Be laboratory neutron source: the ^{239}Pu decay produces α-particle, which is captured by the ^{9}Be nucleus and while emitting a fast neutron, a ^{12}C nucleus is produced.

$$^{9}_{4}Be + ^{4}_{2}\alpha \rightarrow ^{12}_{6}C + ^{1}_{0}n$$

^{239}Pu α $t_{1/2}$ =24300 years neutron yield: 10^5 neutron s^{-1} Bq^{-1}

By their energy, neutrons can be classified:

- Slow neutrons

1. Thermal neutrons 0< E <0, 44 eV

2. Resonance neutrons 0,44< E <1000eV

- Intermediate neutrons 1keV< E <500 keV
- Fast neutrons 0,5 MeV< E < 10 MeV
- High energy neutrons 10 MeV< E < 50 eV
- Ultrafast neutrons 50 MeV< E

Since the (alpha, n-type) laboratory neutron sources practically only provide fast neutrons, the average neutron energy for Pu-Be laboratory neutron source is 4,5 MeV. The neutron energy distribution:

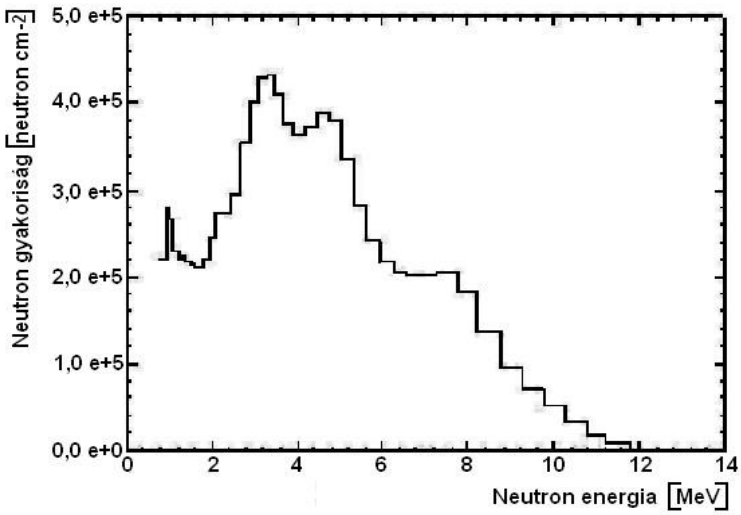

Figure 1.: The neutron energy distribution of the Pu-Be laboratory neutron source

With the fast neutrons the efficiency of the n, gamma-type activation is very poor, therefore the energy of the neutrons is reduced with a moderator by elastic scattering. For this purpose, paraffin is applied, since it has a high proton concentration.

Scheme of the activation device:

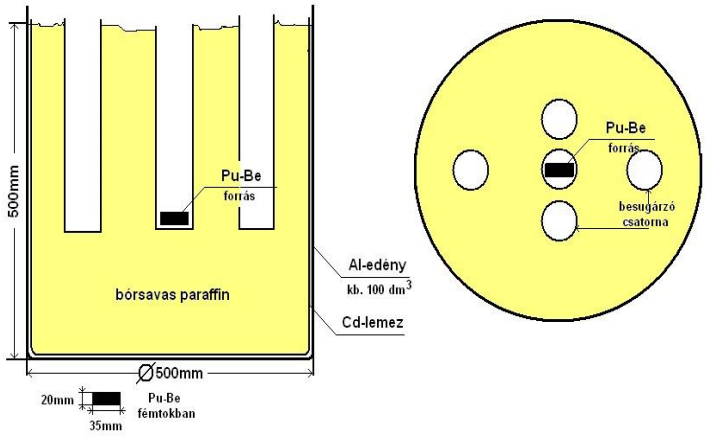

Figure 2.: Pu-Be neutron source

The activation time (t) of the sample can be determined from the following equation:

$$A = \lambda N = \Phi \delta N_T (1 - e^{-\lambda t})$$

where: Φ = neutron flux [n cm^{-2} sec^{-1}]

δ = reaction cross section [barn]

N_T= the number of nucleuses that can be activated

Saturation activity = $\Phi \delta N_T$. Theoretically this state can only be achieved after infinite time ($t \to \infty$). In practice, the proper activity of the sample can be reached after 6-8 times of the (generated) radionuclides half-life. This is due to the radioactive decay starting during activation, after some time an equilibrium will form between the numbers of forming and decaying radioactive nuclides.

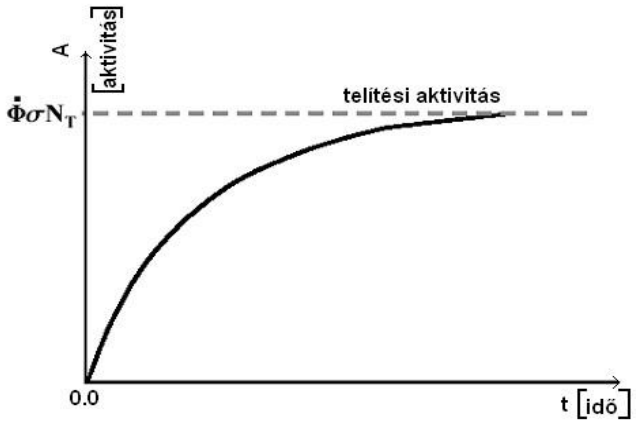

Figure 3.: Saturation activity

Isotopes used during the laboratory practice:

$$^{164}_{66}Dy(n,\gamma)^{165}_{66}Dy \rightarrow (\beta^-,\gamma) \rightarrow ^{165}_{67}Ho$$

A more complex problem is the half-life determination of $^{116m}_{49}In$, since there are two naturally stable isotopes of In which produces 2-2 isotopes during activation:

Table 1.: Indium

Target	Isotope composition [%]	δ_a [barn]*	Product	$t_{/2}$
$^{113}_{49}In$	4,23	56	$^{114m}_{49}In$	49 days
$^{113}_{49}In$	4,23	2	$^{114}_{49}In$	72 sec
$^{115}_{49}In$	95,77	52	$^{116}_{49}In$	14 sec
$^{115}_{49}In$	95,77	155	$^{116m}_{49}In$	54 min

* 1 barn= $10^{-24}cm^2$

From the 4 products we wish to determine the half-life of $^{116m}_{49}In$ therefore, the sample should be cooled (decayed) before the measurements.

If we choose an activation time of 1-3 hours, significant amount of $^{114m}_{49}In$ will not form, since it has a very long half-life compared to the other isotopes. The short half-life isotopes will practically decay with cooling (resting) for about 10 minutes.

$$^{115}_{49}In(n,\gamma)^{116m}_{49}In \rightarrow (\beta^-) \rightarrow ^{116}_{50}Sn$$

In the case of Dy and In samples, the experiment is quite simple, since from one stabile isotope one radioactive isotope is formed, therefore the measurement can be done by measuring the counts every 5 minutes for one minute (with cooling the In sample first).

These experiments are performed with β-scintillation detectors. The gathered data should be transformed to ln (I). The ln (I) values can be plotted with the time values.

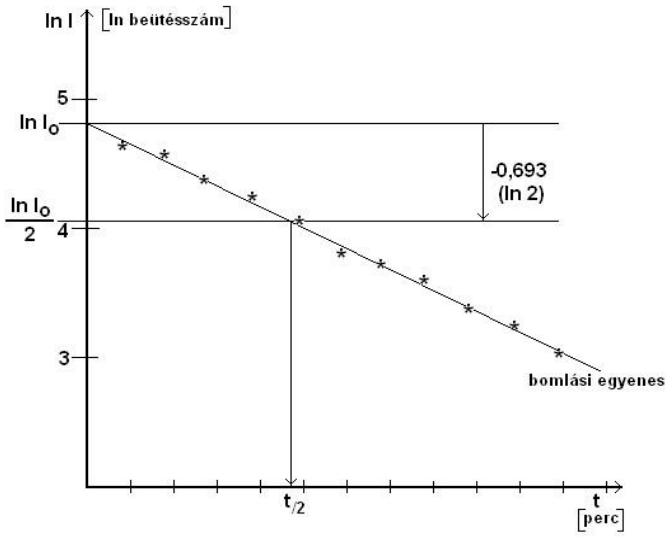

Figure 4.: Half-logarithm graphical transformation

For the analysis we apply half-logarithmic transformation, since our experimental results plot a negative exponential function with time. However, if the counts are transformed, ln (I) – t data couples fit a line, and a line could be defined even from 2 points. Therefore with less data is enough for minimizing the uncertainty related to the statistic behaviour of the radioactive decay.

After plotting the results, the half-life can be calculated graphically, by reading the t value of the (ln (I_0) – ln (2)) value from the line fitted to the measured data. With Excel, the same line can be fitted to the data, and from the line's equation (y = a + bx) the decay constant equals – b (λ = -b), from which the half-life can be calculated. (Excel applies linear regression for calculations).

Silver samples

With the neutron activation of natural silver 3 different radioactive isotopes can be produced. During the experiments, we would like to calculate two isotopes half-life. Silver has two stable isotopes in nature in 50-50%. The determination of the half-lives include a two-step experiment. First, the sample is activated for ~3 minutes, and the counts are measured with a Geiger-Müller detector. The measurement time in this case is 3 seconds, and the computer measures 50 times. From these data, the short half-life isotope can be investigated.

In the next step, the silver is activated for 10-15 minutes, and the measurement time is 10 seconds in this case.

Table 2.: The activity of silver

Target	Isotope composition [%]	δ_a [barn]*	Product	$t_{1/2}$
$^{107}_{47}Ag$	51,839	23	$^{108}_{47}Ag$	2,4 min
$^{109}_{47}Ag$	48,161	55	$^{110}_{47}Ag$	24,2 s
$^{109}_{47}Ag$	48,161	1,56	$^{110m}_{47}Ag$	253 days

If the results are plotted as before, the half-life of $^{110}_{47}$Ag can be determined. If the results are plotted in the second case, the plot will not be linear. This is due to the two isotopes decaying in the same time. The plot can be considered as the sum of two lines. In order to determine the half-life of $^{108}_{47}$Ag, we fir the line for the data after 3 minutes, and calculate the half-life as before.

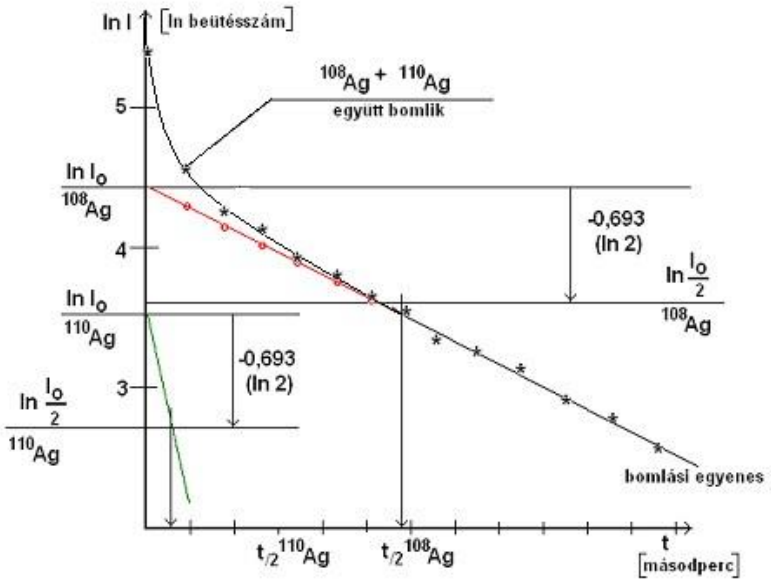

Figure 5.: Ln I – t plot of silver sample

Before each experiment, background radiation should be measured 3-5 times. The length of background measurements should be the same as the measurement times in each case.

Questions:

1. Definition of half-life.
2. The operation of Pu-Be neutron source.
3. Why is a moderator applied during the activation of the samples?
4. How do we estimate the activation time?
5. What is saturation activity?
6. How long does it take to get near to the saturation activity in practice?
7. What parameters should be fixed during the radioactive decay measurements?
8. What detector types are we using for half-life measurements?
9. Why should the Indium samples be cooled (rested) before the experiments?
10. Why is half-logarithm transformation applied for the calculations?